

# A Comparison of Quintessence and Nonlinear Born-Infeld Scalar Field Using Gold Supernova data

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## Abstract

We study the Non-Linear Born-Infeld( NLBI) scalar field model and quintessence model with two different potentials(  $V(\phi) = -s\phi$  and  $\frac{1}{2}m^2\phi^2$ ). We investigate the differences between those two models. We explore the equation of state parameter  $w$  and the evolution of scale factor  $a(t)$  in both NLBI scalar field and quintessence model. The present age of universe and the transition redshift are also obtained. We use the Gold dataset of 157 SN-Ia to constrain the parameters of the two models. All the results show that NLBI model is slightly superior to quintessence model.

**Keywords:**Dark energy; Quintessence Model; NLBI Model.

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## 1 Introduction

According to analyses of CMB[1]+SN-Ia[2]+HST[3]+LSS[4] data, there are strong evidences to indicate that our universe has recently entered a phase of accelerating expansion and that the universe is flat. It implies that if this is not a signal of modifying the standard theory of gravity, that may be a evidence of existing a "exotic homogeneous matter" with negative pressure termed "dark energy"(DE)[5]. An equation of state parameter ( $w = p/\rho$ ) is usually used to describe this energy component. The value of  $w$  is required to less than  $-\frac{1}{3}$  for a accelerating expansion universe. Due to the existence of ordinary matter and radiation,

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we actually need a more negative value of  $w$  to drive accelerating expansion of the universe. Analysis shows  $w$  can lie in the range of  $-1.32 < w < -0.82$  with the  $2\text{-}\sigma$  confidence levels[6]. Up to now a cosmological constant with  $w = -1$  is in good agreement with all the data. However, the result doesn't rule out the scalar field and phantom field models as the dark energy candidate. For the  $\Lambda$ CDM model[7], there are two boring problems: the fine-tuning problem and cosmic coincidence problem[8]. Though same problems remain in other models, it can be alleviated in some models[9].

The scalar field appeared in cosmology is not the first time. As early as more than twenty years ago, theorist normally consider a homogenous scalar field  $\phi$  in an inflationary universe[10]. A scalar field  $\phi$  slowly evolving down its potential  $V(\phi)$  can drive both inflation and late-time accelerating expansion. The standard quintessence scenario[11](a canonical scalar field described by a lagrangian  $L = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ ) is a simplest scalar field. The universes they predict will have significant differences with respect to different potentials. Another more complicated scalar field is K-essence. The idea of K-essence was firstly introduced as a possible model for inflation[12]. Later it was noted that K-essence was introduced as a possible models for dark energy[13]. K-essence can be defined as any scalar field with non canonical kinetic terms. Its lagrangian usually takes the form  $L_A = V(\phi)F(X)$ . A more general form of lagrangian for K-essence[14] is  $L_B = f(\phi)g(X) - V(\phi)$ , where  $\phi$  is the scalar field, and  $X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$ .

The Born-Infeld[15] theory has been considered widely in string theory and cosmology[16]. There are two Born-Infeld type scalar field models. The first is rolling tachyon field[17] with the lagrangian form  $L_{tach} = -V(\phi)\sqrt{1 - \dot{\phi}^2}$ , which can be classified as  $L_A$ . Its interesting features have been widely studied[18]. Another one is the Nonlinear Born-Infeld(NLBI) scalar field theory with the lagrangian form  $L_{NLBI} = \frac{1}{\eta}[1 - \sqrt{1 - \eta\dot{\phi}^2}] - V(\phi)$ (noted, when field velocity  $\dot{\phi} \rightarrow 0, L_{NLBI} = L_{quin} = L = \frac{1}{2}\dot{\phi}^2 - V(\phi)$  by Taylor expansion). Obviously it can be regarded as one form of  $L_B$ . W.Heisenberg proposed this Lagrangian density in order to describe the process of meson multiple production connected with strong field regime[19]. H.P.de Oliveira qualitatively studied the solutions of a three-dimensional dynamical system describing the static and spherically symmetric solutions for this NLBI scalar field [20]. The dark energy model with this NLBI scalar field was recently suggested by H.Q.Lu[21]. However, comparing with tachyon field, the role of NLBI scalar field in cosmology is far beyond study.

One direct thought motivated us to consider the NLBI field is, besides the attractive prop-

erties[19,22], being able to study the role of nonlinearities in the matter fields. Furthermore, we will be able to see whether it can provide more interesting physical results than those generated by ordinary quintessence model.

It is important to choose the potential  $V(\phi)$  for the scalar field. In many cases, we use a potential predicted by particle physics. However, we do not really know which theory of particle physics best describes the universe, we should keep an open mind as to the form of  $V(\phi)$ . One ideal approach is to consider some simple and possible forms of potential, explore the cosmological behavior in detail using the qualitative theory of dynamical systems, and then study the evolution of universe in more detail to quantitatively fit with current observation data. We choose two simple potentials  $-s\phi$  and  $\frac{1}{2}m^2\phi^2$  for further study in this paper. Constraints on the linear potential  $-s\phi$  in quintessence and phantom models from recent supernova data have been argued in[23]. It has been argued[24] that such a potential is favored by anthropic principle considerations and can provide a potential solution to the cosmic coincidence problem. The square potential  $\frac{1}{2}m^2\phi^2$  has been considered in a chaotic inflationary universe[25].

In this paper, we will focus on the differences between NLBI scalar field and linear scalar field(quintessence), explore the cosmological scenario in detail and compare them with SN-Ia Gold data. This paper is organized as follows: In section 2, we simply review the theoretic model with the two scalar fields and derive their cosmological evolution with two different potentials. In section 3, we fit the Hubble parameter to the SN-Ia Gold data, obtain constraint for the potential parameters and compare the NLBI scalar field with the linear scalar field. Section 4 is conclusion and discussion.

## 2 Theoretic Models of Quintessence and NLBI Scalar Field

We consider a spatially homogeneous scalar field in a flat universe with Robertson-Walker metric  $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$ , here we have neglect the relativistic radiation component and assume only presence of non-relativistic matter component and scalar field.

### 1.Linear Scalar Field:

$$L_{quin} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (1)$$

$$\frac{\dot{a}}{a} = \frac{1}{\sqrt{3}Mp} \left[ \frac{1}{2}\dot{\phi}^2 - V(\phi) + \rho_0 \frac{a_0^3}{a^3} \right]^{\frac{1}{2}} \quad (2)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \quad (3)$$

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \quad (4)$$

$$c_s^2 = 1 \quad (5)$$

## 2.NLBI Scalar Field:

$$P_{NLBI} = L_{NLBI} = \frac{1}{\eta}[1 - \sqrt{1 - \eta\dot{\phi}^2}] - V(\phi) \quad (6)$$

$$\rho_{NLBI} = \frac{1}{\eta\sqrt{1 - \eta\dot{\phi}^2}} - \frac{1}{\eta} + V(\phi) \quad (7)$$

$$\frac{\dot{a}}{a} = \frac{1}{\sqrt{3}Mp} \left[ \frac{1}{\eta\sqrt{1 - \eta\dot{\phi}^2}} - \frac{1}{\eta} + V(\phi) + \rho_0 \frac{a_0^3}{a^3} \right]^{\frac{1}{2}} \quad (8)$$

$$\ddot{\phi} + (1 - \eta\dot{\phi}^2) \left[ 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} (1 - \eta\dot{\phi}^2)^{\frac{1}{2}} \right] = 0 \quad (9)$$

$$w = \frac{1 - \sqrt{1 - \eta\dot{\phi}^2} - \eta V(\phi)}{\frac{1}{\sqrt{1 - \eta\dot{\phi}^2}} - 1 + \eta V(\phi)} \quad (10)$$

$$c_s^2 = 1 - \eta\dot{\phi}^2 \quad (11)$$

We can see from above equations, both the field equation(Eq.(9)) and sound speed of NLBI scalar field(Eq.(11)) will recover to quintessence model(Eqs.(3,5) if  $\dot{\phi} \rightarrow 0$ . This is consistent with above description of NLBI lagrangian. No matter for quintessence or NLBI scalar field it is worth noting that potential( $V(\phi)$ ) term can not determine whether w crosses the phantom divide line(PDL),it can not directly determine the value of sound speed too. Additionally, When the potential  $V(\phi) = \frac{1}{\eta}$ , Eq.(6) actually describes the lagrangian of Chaplygin gas[26]

One apparent effect of DE is through its impact on the expansion rate, which is determined by the equation of state of the DE component. It gives us an opportunity to explore the information of DE by measurements of the relationship between luminosity distance and redshift  $z$ . Since quintessence and NLBI scalar field have different evolution of the equation of state  $w$ , It provides us a possibility to distinguish between quintessence and NLBI scalar field by comparing with SN-Ia data and future SNAP data. Another effect of DE is directly through the perturbation to affect the CMB. This feature inform us another opportunity to test DE. It is by looking at the anisotropy in the temperature and polarization of CMB, which have been measured by WMAP and will be PLANCK mission. The speed of sound of DE

mainly affects the CMB spectrum at largest scales via the late-time Integrated Sachs Wolfe effect[27]. Since  $c_s^2$  determines how fast fluctuations dissipate, a lower sound speed increases the phase space of models which are Jeans unstable[28]. Hence the different value of sound speed predicted by quintessence model and NLBI scalar field model( $c_s^2 = 1$  for quintessence and  $c_s^2 = 1 - \eta\dot{\phi}^2$  for NLBI scalar field) makes the spectra of anisotropies of the two models slightly different because of the different Jeans scales in the DE sector. So we have another possibility to distinguish between quintessence and NLBI scalar field by analysing the WMAP data and other observational data. In this paper, we focus our interest on SN-Ia measurement and remain the latter possibility for further study.

### 3 The Evolution of Universe of Quintessence and NLBI scalar Field with Two Special Potentials

#### A. $V(\phi) = -s\phi$

By setting  $H_0 t \rightarrow t, \phi \rightarrow \sqrt{3}M_p\phi, s \rightarrow \sqrt{3}H_0^2 s$ , the evolution equation of scale factor and the scalar field equation for quintessence model can be written in rescaled form as

$$\frac{\dot{a}}{a} = [\frac{1}{2}\dot{\phi}^2 - s\phi + \frac{\Omega_{om}}{a^3}]^{\frac{1}{2}} \quad (12)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - s = 0 \quad (13)$$

By the same transformation, the corresponding equations for NLBI model are

$$\frac{\dot{a}}{a} = [\frac{1}{\sqrt{1 - \eta\dot{\phi}^2}} - 1 - s\phi + \frac{\Omega_{om}}{a^3}]^{\frac{1}{2}} \quad (14)$$

$$\ddot{\phi} + (1 - \dot{\phi}^2)[3\frac{\dot{a}}{a}\dot{\phi} + s(1 - \dot{\phi}^2)^{\frac{1}{2}}] = 0 \quad (15)$$

where we set  $\eta = 1/\rho_c = 1/3M_p^2 H_0^2$ . We follow Ref[23] and solve the system numerically using the following initial conditions( $t \rightarrow t_i \simeq 0$ ):  $a(t_i) = (\frac{9\Omega_{0m}}{4})^{1/3} t_i^{2/3}$ ,  $\dot{\phi}(t_i) = 0$ ,  $\phi(t_i) = \phi_i$ . The numerical program[29] is modified to be suited for us. The value of  $\phi_i$  is chosen for each  $s$  such that  $\Omega_{0\phi} = \frac{1}{\sqrt{1 - \dot{\phi}(t_0)^2}} - 1 - V(\phi(t_0)) = 1 - \Omega_{0m}$  at the present time  $t_0$ (for quintessence model  $\Omega_{0\phi(t_0)} = \frac{1}{2}\dot{\phi}^2(t_0) + V(\phi(t_0))$ ). We set  $a(t_0) = 1$  and assume a prior of  $\Omega_{0m} = 0.3$ .

From Eq.(3), we can easily obtain

$$\rho_{quin} = -3 \int H \dot{\phi}^2 dt + const \quad (16)$$

We can also obtain a similar equation for NLBI scalar field:

$$\rho_{NLBI} = -3 \int H \frac{\dot{\phi}^2}{\sqrt{1 - \eta \dot{\phi}^2}} dt + const \quad (17)$$

These two equations inform that only in a static universe, the energy density of scalar field is unchanged. In a expansive(contractive) universe  $H > 0 (H < 0)$ , the energy density of scalar field rolls down(up) the potential for both quintessence and NLBI scalar field model. The numerical results are plotted in fig1-4.

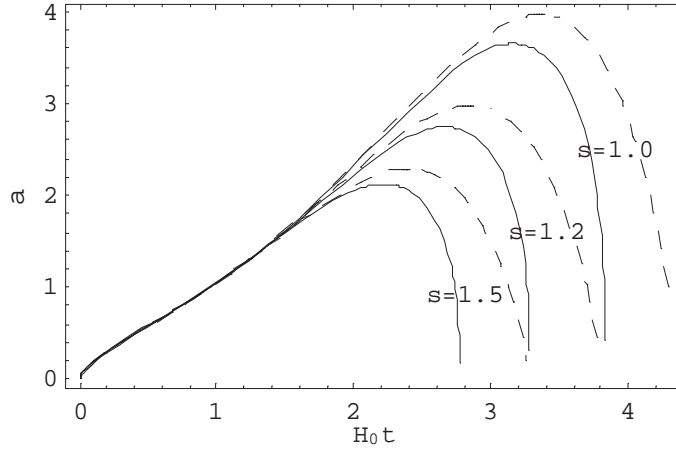


Fig1. The evolution of scalar factor for quintessence field and NLBI scalar field with linear negative potential. Solid line for quintessence, dot line for NLBI scalar field. The time that universe begins to contract in NLBI scalar field is later than that in quintessence models. This feature is shown clearly in fig4.

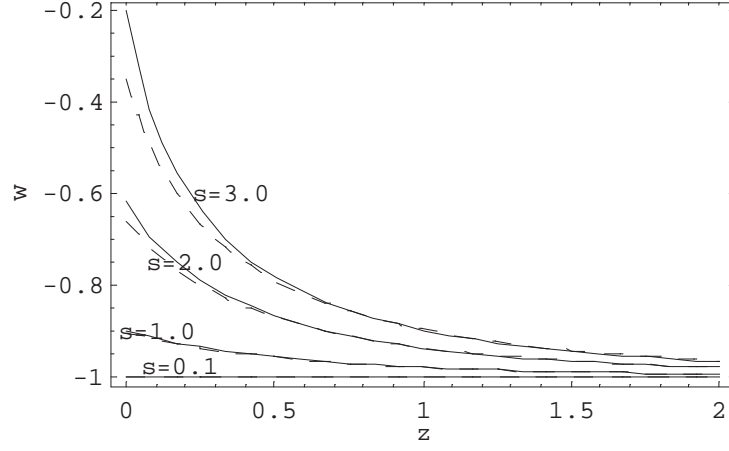


Fig2. The evolution of the equation of state  $w(z)$  with respect to redshift  $z$ . Solid line for quintessence, dot line for NLBI scalar field. The result shows that a low value of parameter  $s$  (therefore a low value of  $w(z)$ ) is favored by current observation. For the same parameter  $s$ , NLBI model has a lower value of equation of state  $w$ , but the difference is not significant when  $s$  is close to zero. The equation of state  $w$  can not cross the PLD for both quintessence and NLBI model.

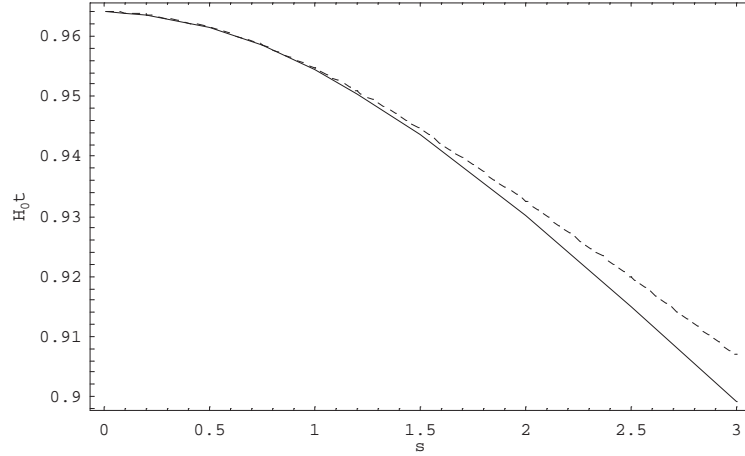


Fig3. The present age of universe  $H_0 t_0$  with different  $s$ . Solid line for quintessence, dot line for NLBI scalar field. The curve has been sampled at  $s = 0.01, 0.1, 0.2, 0.5, 0.75, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0$  and the corresponding points have been joined. The result is consistent with observation to globular cluster and white dwarf[30]. For  $s = 0.1$ , the age of universe will be 13.77 Gyr for NLBI scalar field model if we take  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

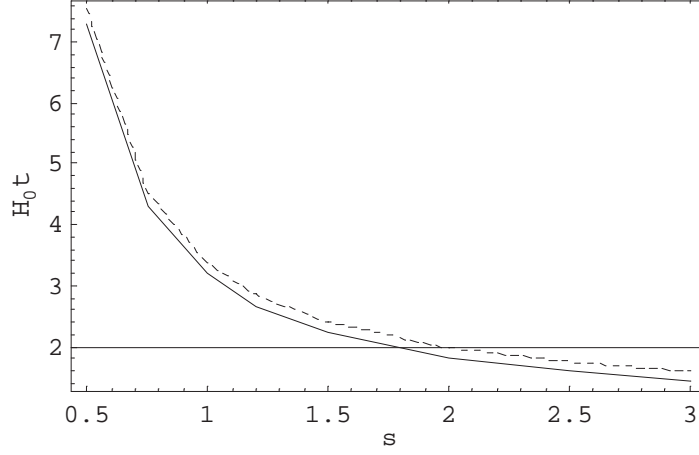


Fig4. The age of universe  $H_0 t$  with different  $s$  when scale factor starts to contract. Solid line for quintessence, dot line for NLBI scalar field. The curve has been sampled at  $s = 0.01, 0.1, 0.2, 0.5, 0.75, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0$  and the corresponding points have been joined. It is worth noting from fig3 and fig4 that the universe with NLBI scalar field has longer age than the universe with quintessence.

**B.**  $V(\phi) = \frac{1}{2}m^2\phi^2$

By setting  $H_0 t \rightarrow t, \phi \rightarrow \sqrt{3}M_p\phi, m \rightarrow mH_0$ , we can obtain the equations for quintessence:

$$\frac{\dot{a}}{a} = \left[ \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 + \frac{\Omega}{a^3} \right]^{\frac{1}{2}} \quad (18)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - m^2\phi = 0 \quad (19)$$

By the same transformation, the corresponding equations for NLBI model are

$$\frac{\dot{a}}{a} = \left[ \frac{1}{\sqrt{1 - \eta\dot{\phi}^2}} - 1 + \frac{1}{2}m^2\phi^2 + \frac{\Omega}{a^3} \right]^{\frac{1}{2}} \quad (20)$$

$$\ddot{\phi} + (1 - \dot{\phi}^2) \left[ 3\frac{\dot{a}}{a}\dot{\phi} + m^2\phi(1 - \dot{\phi}^2)^{\frac{1}{2}} \right] = 0 \quad (21)$$

We numerically solve the equations with same conditions as negative linear potential.



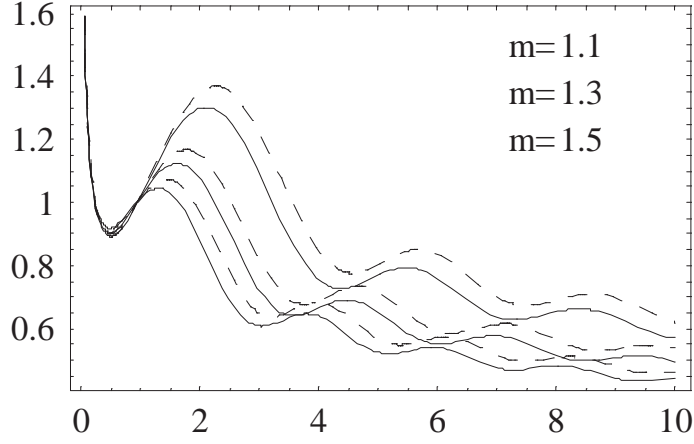


Fig5. The evolution of  $\dot{a}$  with respect to  $H_0 t$ . The horizontal axis represents  $H_0 t$  and the vertical axis represents  $\dot{a}$ . Solid line for quintessence, dot line for NLBI scalar quintessence. Unlike negative potential, the universe with a positive potential will expand for ever. For square potential, the numerical result indicates that our universe has entered a phase of accelerated expansion from a phase of decelerated expansion in the recent past. This is consistent with observational result[31]. Furthermore, the result also shows that accelerated expansion and the decelerated expansion will appear by turns.

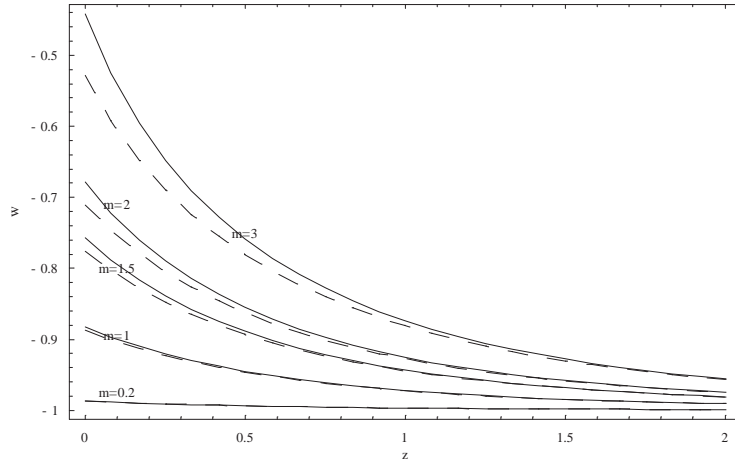


Fig6. The evolution of the equation of state  $w(z)$  with respect to redshift  $z$ . Solid line for quintessence, dot line for NLBI scalar quintessence. The result shown here is similar to that in fig2.

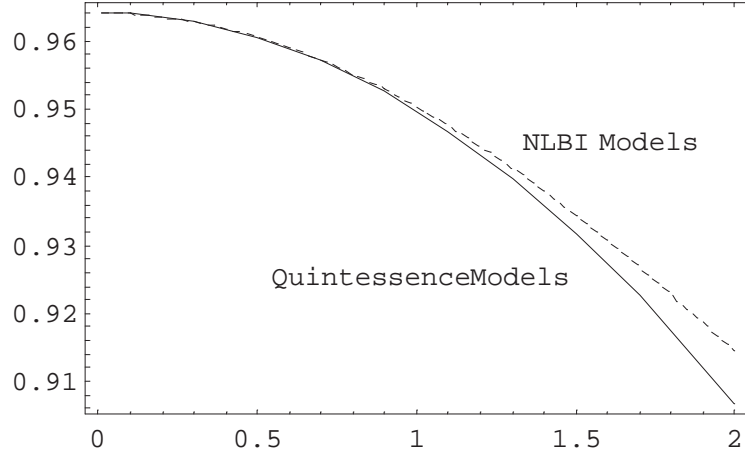


Fig7. The age of current universe  $H_0 t$  with different  $m$ . The horizontal axis represents  $m$  and the vertical axis represents  $H_0 t$ . The result has no significant difference with that in fig3. The curve has been sampled at  $m = 0.01, 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 2.0$  and the corresponding points have been jointed.

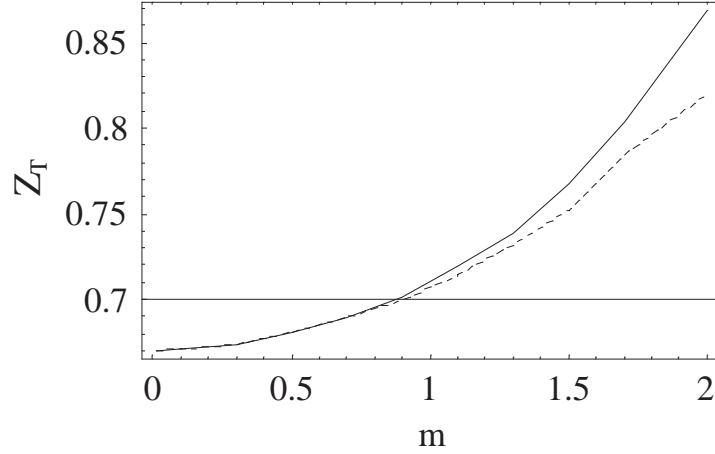


Fig8. The transition redshift  $Z_T$  ( the redshift that universe evolves from decelerating expansion to accelerating expansion) with different  $m$ . The result does not conflict with the result in Ref[31]. The curve has been sampled at  $m = 0.01, 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 2.0$  and the corresponding points have been jointed.

## 4 Fit to the SN-Ia Gold Data Set

We use the SN-Ia observations to put constraints on the quintessence and NLBI models. To do this, we need to obtain the corresponding Hubble parameter  $H(z, s \& m) = \frac{\dot{a}}{a}(z, s \& m)$  as a

function of redshift. The luminosity distance( $d_L$ ) for a source at redshift  $z$  located at radial coordinate distance  $r$  is given by  $d_L = (1+z)a_0r$  where  $a_0$  is the present value of the scale factor. Then we can obtain the corresponding Hubble free luminosity distance in a spatially flat expanding universe:

$$D_L^{th}(z, s\&m) = H_0 d_L = H_0(1+z) \int_0^z \frac{dz'}{H(z, s\&m)} \quad (22)$$

The apparent magnitude is connected to  $D_L(z)$  as

$$m^{th}(z, s\&m) = M' + 5\log_{10}(D_L^{th}(z, s\&m)) \quad (23)$$

where

$$M' = M + 5\log_{10}\left(\frac{cH_0^{-1}}{Mpc}\right) + 25 \quad (24)$$

The best fitting values of the parameters can be obtained through  $\chi^2$  minimization, where

$$\chi^2 = \sum_{i=1}^{157} \frac{[m^{obs}(z_i) - m^{th}(z_i, s\&m)]^2}{\sigma_{m^{obs}(z_i)}^2} \quad (25)$$

The corresponding observed  $D_L^{obs}(z_i)$  ( $i = 1, \dots, 157$ ) comes from the Gold SN-Ia data set.

The observational data are given as the apparent magnitudes  $m^{obs}(z)$  and  $1\sigma$  errors  $\sigma_{m^{obs}(z_i)}$ .

We marginalize the nuisance parameter  $M'$  by defining a new  $\chi'^2$ :

$$\chi'^2 = -2\ln \int_{-\infty}^{+\infty} e^{-\chi^2/2} dM' \quad (26)$$

We finally obtain the effective  $\chi^2(s\&m)$ :

$$\chi^2(s\&m) = \chi^2(M' = 0, s\&m) - \frac{B(s\&m)}{C} \quad (27)$$

where

$$B(s\&m) = \sum_{i=1}^{157} \frac{[m^{obs}(z_i) - m^{th}(z_i, M' = 0, s\&m)]^2}{\sigma_{m^{obs}(z_i)}^2} \quad (28)$$

$$C = \sum_{i=1}^{157} \frac{1}{\sigma_{m^{obs}(z_i)}^2} \quad (29)$$

Minimizing  $\chi^2(s\&m)$  we can find the best fit value of parameter  $s\&m$  ( $\chi^2(s_0\&m_0) = \chi_{min}^2$ ).

The  $1\sigma$  error on one parameter is determined by the relation  $\Delta\chi^2_{1\sigma} = \chi^2_{s\&m} - \chi^2_{min} = 1$ , which means the parameter  $s\&m$  in the range  $[s_0, s_{1\sigma}][m_0, m_{1\sigma}]$  with 68% probability. For  $2\sigma$  error(95.4% range)  $\Delta\chi^2_{2\sigma} = 4$  and  $3\sigma$  error(99% range)  $\Delta\chi^2_{3\sigma} = 6.63$ .

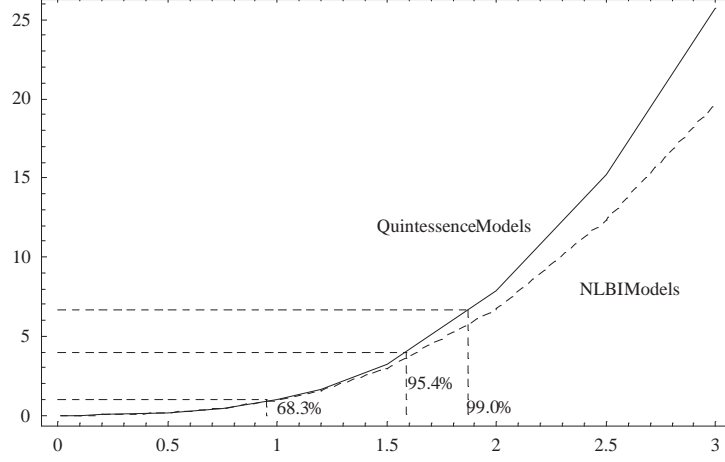


Fig9. The differences  $\Delta\chi^2 = \chi^2_s - \chi^2_{s\approx 0}$  for negative linear potential  $-s\phi$ . The horizontal axis represents  $s$  and the vertical axis represents  $\Delta\chi^2$ . Solid line for quintessence, dot line for NLBI scalar field. The curve has been sampled at  $s = 0.01, 0.1, 0.2, 0.5, 0.75, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0$  and the corresponding points have been joined.

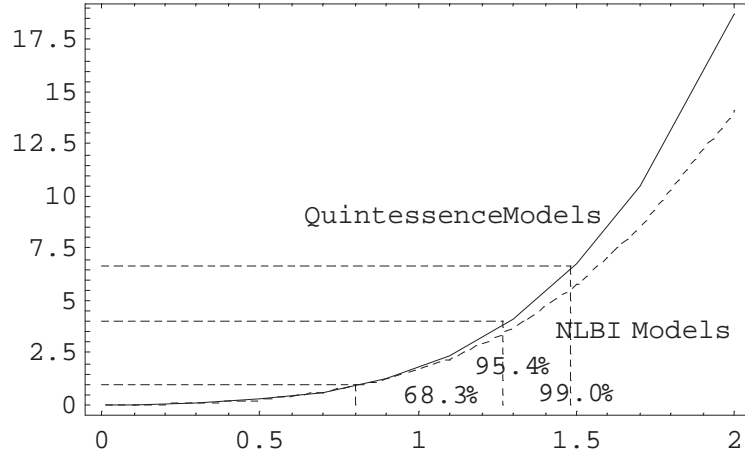


Fig10. The differences  $\Delta\chi^2 = \chi^2_m - \chi^2_{m\approx 0}$  for square potential  $\frac{1}{2}m^2\phi^2$ . The horizontal axis represents  $m$  and the vertical axis represents  $\Delta\chi^2$ . Solid line for quintessence, dot line for NLBI scalar field. The curve has been sampled at  $m = 0.01, 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 2.0$  and the corresponding points have been joined.

Fig9 and Fig 10 show plots of the difference  $\Delta\chi^2$  with respect to the cosmological constant ( $\chi^2(s \simeq 0) = \chi^2(m \simeq 0) = 177.1$ ). It is implied from fig9 and fig10 that the  $\chi^2$  value of NLBI scalar field is lower than the value of quintessence for the same  $s$  &  $m$  value. This give us a positive information that NLBI scalar field model may be superior to quintessence model.

## 5 Conclusion and Discussion

Through the analyses of the two potentials in quintessence model and NLBI model, we can conclude that:

Firstly, the principle we choose the potential is that the theoretical prediction should be consistent with the observational universe (such as the observation of universe age, the CMBR measurement, the SN-Ia observation, the structure formation and so on). It is to say the predicted universe need to have nearly same "history" ( to account for the observational data), but the fate of the predicted universe could have significant differences.

Secondly, for non-negative potentials, the common feature of the further universe is that they will continue expanding for ever, though the fate of the universes with different potentials may be dramatically altered.

Thirdly, if the potential can evolve into negative value, the universe will evolve continually from expansion ( $H > 0$ ) to contraction ( $H < 0$ ). This is not the case for positive potential (see fig1 and fig5). The cosmology with negative potential has been discussed in Ref[32].

Finally, comparing our theoretic models with the observational data of SN-Ia, the age of universe  $H_0 t$ , the equation of state  $w$  and the transition redshift  $z$ , we can conclude that the NLBI modes is consistent with all the observations. Furthermore the result shows that the NLBI model slightly excels quintessence model. However, the result also shows that a smaller value of  $s$  &  $m$  (a slower rolling of field  $\phi$  correspondingly ) can provide better fits with observational data, but in this case the difference between NLBI scalar field and quintessence is not distinct. In order to get a more convincible result, the effect of NLBI scalar field in CMB (for instance the late-time ISW effect) should be studied carefully.

## 6 Acknowledgement

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## References

- [1]P.de Bernardis et al., Nature(London) **404**:955(2000);  
S Hannany et al., Astrophys.J.**545** L1-L4(2000).
- [2]A.G.Riess et al., Astron.J.**116**:1009(1998);  
S.Perlmutter et al., Astrophys.J.**517**:565(1999);  
P.M.Garnavich et al., Astrophys.J. **509**:74(1998);  
J.L.Shievers et al., Astrophys.J.**591**:590(2003).
- [3]W.Freedman et al., Astrophys.J.**553**:47(2001).
- [4]M.Tegmark et al.,(SDSS Collaboration), Phys.Rev.D**69**:103501(2004);  
M.Tegmark et al.,(SDSS Collaboration), Astrophys.J.**606**:702(2004);  
M.Colless et al., astro-ph/0306581.
- [5]N.Bahcall, .J.P.Ostriker, S.Perlmutter and P.J.Steinhardt, Science **284**:1481(1999).
- [6]A.Melchiorri, astro-ph/0406652.
- [7]S.Weinberg, Rev.Mod.Phys**61**(1):1(1989);  
P.J.E.Peebles, B.Ratra, Rev.Mod.Phys**75**(2):559(2003);  
G.W.Gibbons, hep-th/0302199
- [8]P.J.Steinhardt,L.Wang and I.Zlatev, Phys.Rev.D**59**:123504(1999)  
P.Steinhardt, in *Critical Problems in Physics*, edited by V.L.Fitch and D.R.Marlow(Princeton University Press, Princeton, NJ(1997)
- [9]I.Zlatev, L.Wang and P.J.Steinhardt, Phys.Rev.Lett**82**:896(1999);  
P.Singh, M.Sami and N.Dadhich, Phys.Rev.D**68**:023522(2003);  
J.G.Hao and X.Z.Li, Phys.Rev.D**67**:107303(2003);  
J.G.Hao and X.Z.Li, Phys.Rev.D**68**:043501(2003);  
D.J.Liu and X.Z.Li, Phys.Rev.D**68**:067301(2003);  
T.Chiba, Phys.Rev.D**66**:063514(2002);  
L.Amendola and D.Tocchini-Valentini, Phys.Rev.D**64**:043509(2001);  
X.Z.Li, J.G.Hao and D.J.Liu, Chin.Phys.Lett**19**:1584(2002);  
G.Huey and B.D.Wandelt, astro-ph/0407196;  
W.Fang, H.Q.Lu, Z.G.Huang and K.F.Zhang, hep-th/0409080(To appear in Int.J.Mod.Phys.D).

- [10] P.J.E.Peebles and B.Ratra, *Astrophys.J.Lett***325**:L17(1988).
- [11] C.Wetterich, *Nucl.Phys.B***302**:668(1988);  
B.Ratra and P.J.E.Peebles, *Phys.Rev.D***37**:3406(1988);  
R.R.Caldwell,R.Dave and P.J.Steinhardt,*Phys.Rev.Lett***80**:1582(1998).
- [12] C.Armendáriz-Picón,T.Damour and V.Mukhanov,*Phys.Lett.B***458**:209(1999);  
J.Garriga and V.F.Mukhanov.*Phys.Lett.B***458**:219(1999);  
A.G.Riess, astro-ph/0402512; L.P.Chimento,astro-ph/0311613.
- [13] C.Armendáriz-Picón, V.Mukhanov and P.J.Steinhardt,*Phys.Rev.Lett***85**:4438(2000);  
C.Armendáriz-Picón, V.Mukhanov and P.J.Steinhardt,*Phys.Rev.D***63**:103510(2001);  
T.Chiba, *Phys.Rev.D***66**:063514;  
T.Chiba, T.Okabe and M.Yamaguchi, *Phys.Rev.D***62**:023511(2000);  
M.Malquarti, E.J.Copeland, A.R.Liddle and M.Trodden, *Phys.Rev.D***67**:123503(2003);  
R.J.Sherrer, *Phys.Rev.Lett.***93**:011301(2004);  
L.P.Chimento, *Phys.Rev.D***69**:123517(2004)
- [14] A.Melchiorri, L.Mersini,C.J.Odman and M.Trodden, *Phys.Rev.D***68**:043509(2003).
- [15] M.Born and Z.Infeld, *Proc.Roy.Soc A***144**:425(1934).
- [16] G.W.Gibbons and C.A.R.Herdeiro, *Phys.Rev.D***63**:064006(2001);  
G.W.Gibbons *Rev.Mex.Fis.***49S1**:19-29(2003)(hep-th/0106059);  
V.V.Dyadichev, D.V.Gal'tsov and A.G.Zorin,*Phys.Rev.D***65**:084007(2002);  
D.N.Vollick, *Gen.Rel.Grav.***35**:1511-1516(2003); gr-qc/0506091
- [17] A.Sen, *JHEP***0207**:065(2002); *JHEP***0204**:048(2002); *Mod.Phys.Lett.A***17**:1797(2002);  
G.W.Gibbons, *Phys.Lett.B***537**:1(2002); *Glass.Quant.Grav.***20**:S321-S346(2003).
- [18] L.R.Abramo, F.Finelli and T.S.Pereira, *Phys.Rev.D***70**:063517(2004);  
J.S.Bagla, H.K.Jassal and T.Padmanabhan, *Phys.Rev.D***67**:063504(2003).
- [19] W.Heisenberg, *Z.Phys.***133**:79(1952); **126**:519(1949); **113**:61(1939).
- [20] H.P.de Oliveira, *J.Math.Phys.***36**:2988(1995).
- [21] H.Q.Lu, *Int.J.Modern.Phys.D***14**:355(2005); H.Q.Lu et al. hep-th/0504038.
- [22] T.Taniuti,*Prog.Theor.Phys.(kyoto) Suppl***9**:69(1958).
- [23] L.Perivolaropoulos, *Phys.Rev.D***71**:063503(2005).
- [24] J.Garriga and A.Vilenkin, *Phys.Rev.D***61**:083502(2000); *Phys.Rev.D***67**:043503(2003);  
J.Garriga A.Linde and A.Vilenkin, *Phys.Rev.D***69**:063521(2004);  
P.P.Avelina, *Phys.Lett.B***611**:15(2005).

- [25] A.D.Linde, Phys.Lett.B**129**:177(1983).
- [26] A.Kamenshchik, U.Moschella and V.Pasquier, Phys.Lett.B**511**:265-268(2001);  
N.Bilic, G.B.Tupper and R.D.Viollier, Phys.Lett.B**535**:17-21(2002);  
M.C.Bento, O.Bertolami and A.A.Sen, Phys.Rev.D**70**:083519(2004);  
M.C.Bento, O.Bertolami and A.A.Sen, Phys.Rev.D**66**:043507,(2002);  
M.C.Bento, O.Bertolami and A.A.Sen, Gen.Rel.Grav.**35**:2063-2069(2003);  
M.C.Bento, O.Bertolami and A.A.Sen, Phys.Lett.B**575**:172-180,(2003);  
M.C.Bento, O.Bertolami and A.A.Sen, Phys.Rev.D**67**:063003(2003);  
P.T.Silva and O.Bertolami, Astrophys.J.**599**:829-838(2003);  
O.Bertolami, A.A.Sen, S.Sen and P.T.Silva, Mon.Not.Roy.Astron.Soc.**353**:329(2004);  
M.C.Bento, O.Bertolami, N.M.C.Santos and A.A. Sen, Phys. Rev.D**71**:063501(2005);  
N.Bilic, G.B.Tupper and R.D.Viollier, astro-ph/0207423;  
J.C.Fabris, S.V.B.Goncalves and P.E.de Souza, astro-ph/0207430;  
A.Dev, J. S.Alcaniz and D.Jain, Phys.Rev.D**67**:023515(2003);  
V.Gorini, A.Kamenshchik and U.Moschella, Phys.Rev.D**67**:063509(2003);  
M.Makler, S.Q.de Oliveira and I.Waga, Phys.Lett.B**555**:1(2003);  
R.Colistete Jr., J.C.Fabris, S.V.B.Goncalves and P.E.de Souza, gr-qc/0210079;  
D.j.Liu and X.Z.Li, Chin.Phys.Lett.**22**:1600-1603(2005);  
X.Zhang, F.Q.Wu and J.F.Zhang, astro-ph/0411221;  
Y.G.Gong, JCAP0503:007(2005);  
F.Perrotta, S.Matarrese and M.Torki, Phys.Rev.D**70**: 121304(2004);  
V.Gorini, A.Kamenshchik, U.Moschella and V.Pasquier, gr-qc/0403062;  
J.S.Alcaniz and J.A.S.Lima, Astrophys.J.**618**:16(2005);  
N.Bilic, R.J.Lindebaum, G.B.Tupper and R.D.Viollier, JCAP0411:008(2004);  
J.V.Cunha, J.S.Alcaniz and J.A. S.Lima, Phys.Rev.D**69**:083501(2004).
- [27] J.K.Erickson, R.R.Caldwell, P.J.Steinhardt, C.Armendáriz-Picón and V.Mukhanov, Phys.Rev.Lett**88**:121301(2002)  
S.Dedeo, R.R.Caldwell and P.J.Steinhardt, Phys.Rev.D**67**:103509(2003).
- [28] R.Bean and O.Dore, Phys.Rev.D**69**:083503(2004).
- [29] [http:// leandros.physics.uoi.gr/negpot.htm](http://leandros.physics.uoi.gr/negpot.htm).
- [30] E.Carretta et al., Astrophys.J**533**:215(2000);  
W.L.Freedman et al., Astrophys.J**553**:47(2001);  
H.B.Richer et al., Astrophys.J**574**:L15(2002);  
B.M.S.Hansen et al., Astrophys.J**574**:L155(2002);  
L.Krauss, Astrophys.J**596**:L1(2003); J.Cepa, Astron.Astrophys.**422**:831(2004).



- [31] A.G.Riess, *Astrophys.J***560**:49(2001);  
M.S.Turner and A.G.Riess, *Astrophys.J***569**:18(2002);  
R.A.Daly and S.G.Djorgovski, *Astrophys.J***597**:9(2003);  
Y.G.Gong and C.K.Duan, *Class.Quantum.Grav.***21**:3655(2004).
- [32] G.Felder, A.Frolov, L.Kofman and A.Linde, *Phys.Rev.D***66**:023507(2002).